A Reduction of Interferometric Phase-to-Intensity Conversion Noise in Fiber Links by Large Index Phase Modulation of the Optical Beam

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***Abstract*— A noise reduction scheme for long haul fiber amplitude modulation (AM) systems is proposed and analyzed. Such systems suffer from intensity noise which results from interference between the (twice) Rayleigh scattered light and the directly transmitted beam. This interference converts the fundamental phase noise of the laser to intensity noise. We show that a strong phase modulation of the output of the laser beam causes large reduction of the detected signal noise in the vicinity of the detected signal components.**

I. Introduction

It is well known by now that the interference between light which has been retroreflected twice by Rayleigh scattering and the main beam converts the output phase fluctuations of the laser field to intensity fluctuations at the output of a long ($> 1/\alpha$, where α is the loss coefficient) fiber [2]–[4]. The spectrum of this noise consists of a base band extending from zero to roughly twice the laser linewidth $(\Delta\nu)_{\text{laser}}$. If the laser is modulated at frequencies exceeding $(\Delta\nu)_{\text{laser}}$, then the spectrum of the detected intensity will consists of similar noise pedestals straddling each modulation frequency due to beating between information sidebands and the carrier. This in turn degrades the signal-to-noise ratio of the detected AM signal. The analysis which follows shows how the detected signal-to-noise ratio at the output of long fibers can be improved by strong phase modulation of the signal prior to launching into the fiber.

Consider a laser diode (LD) operating continuously (CW) without internal modulation. The message (signal) one desires to transmit is encoded by external amplitude modulation (AM) at ω_m (typically about a few hundred megahertz). Normally then the laser beam is coupled into a long single-mode fiber in an optical telecommunication system. As a way to reduce intensity fluctuations at the output end of fiber because of mixing between direct transmitted light and Rayleigh scattered light, an external phase modulation is introduced at Ωm ($>> \omega_m$). See Fig. 1.

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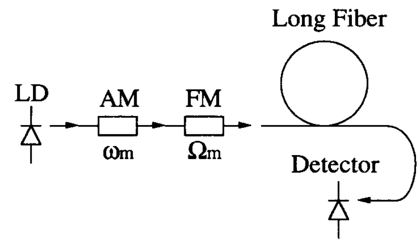

Fig. 1. The optical system under study: LD operating at CW, signal encoded by AM modulation at ω_m, FM modulation at very high frequency Ω_m, laser beam transmitted through long fiber, and finally detected.

The analytic signal of the laser field is given by

$$E(0,t) = E_0(1 + m\cos\omega_m t)e^{i[\omega t + \delta\cos\Omega m t + \phi(t)]}$$
$$\Omega_m >> \omega_m \tag{1}$$

where ω is the average optical frequency, $\phi(t)$ is the random (noise) phase due to the spontaneous emission, and E_0 is the amplitude (whose fluctuations are neglected), m is the (information) AM modulation index, ω_m is the AM modulation frequency, δ is the phase modulation index, Ω_m is the phase modulation frequency. The phase noise can be characterized by [1]

$$\langle \Delta\phi(t_1)\Delta\phi(t_2)\rangle = \frac{2}{\tau_c}\min(t_1, t_2), \quad t_1, t_2 \geq 0 \tag{2}$$

where, $\Delta\phi(t) = \phi(t) - \phi(0)$, and τ_c is the coherence time of the laser.

The single-mode fiber is modeled as made up of $N(\rightarrow\infty)$ sections with effective indexes of refraction $n_1, n_2, \cdots, n_N$ (see Fig. 2). The Rayleigh scattering in optical fiber is due to the small random index inhomogeneities whose statistics are described by the correlation relationship

$$\langle \Delta n(z)\Delta n(z')\rangle = \beta^2 \exp\{-|z - z'|/z_R\} \tag{3}$$

where, $\Delta n(z) = n(z) - n_0$, $n(z)$ is the index of refraction at z, n_0 is the average index of refraction in the fiber, $\langle \Delta n^2(z)\rangle = \beta^2 << 1$, and z_R is the coherence length of the index fluctuations.

The total field at the detector is the sum of the "direct" beam and the doubly scattered beam

$$E_T(L,t) = E_0(L,t) + E_{ds}(L,t) \tag{4}$$

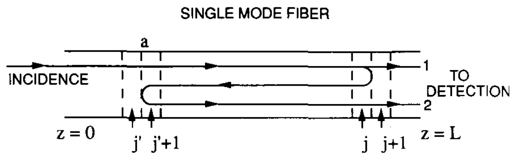

Fig. 2. A single-mode fiber made up of N ($\to \infty$) sections of length $a = L/N$ ($\to 0$). Beam 1 and 2 are direct transmitted and twice Rayleigh backscattered (first at $z = ja$, then at $z = j'a$) light, respectively. The total field at $z = L$ is the sum of that of beam 1 and 2.

and,

$$E_0(L,t) = E_0(1 + m\cos\omega_m t)e^{-\alpha L/2}e^{i[\omega t + \delta\cos\Omega_m t + \phi(t)]} \tag{5}$$

$$\begin{aligned} E_{ds}(L,t) = {} & E_0\, e^{i\omega t}e^{-\alpha L/2}\left(-\frac{1}{4n_0^2}\right)\sum_{j=1}^{L/a}\sum_{j'=0}^{j-1} \\ & \cdot(\Delta n_{j+1} - \Delta n_j)(\Delta n_{j'+1} - \Delta n_{j'}) \\ & \cdot\left[1 + m\cos\omega_m\left(t - \frac{2(j-j')a}{v}\right)\right]e^{-(\alpha+2ik)(j-j')a} \\ & \cdot e^{i\phi\left[t - \frac{2(j-j')a}{v}\right]}e^{i\delta\cos\Omega_m\left[t - \frac{2(j-j')a}{v}\right]} \end{aligned} \tag{6}$$

where, α is the loss coefficient, L is the fiber length, $a = L/N$ ($\to 0$), $k = n_0\omega/c$ is the propagation constant of laser mode in the fiber, and v is the modal group velocity.

The photocurrent at the detector is proportional to the product of $E_T(L,t)$ and its complex conjugate

$$i_d(t) = s_d I_0 e^{-\alpha L}\left\{(1 + m\cos\omega_m t)^2 + g(t) + g^*(t)\right\} \tag{7}$$

and

$$\begin{aligned} g(t) = {} & -\frac{1}{4n_0^2}\sum_{j=1}^{L/a}\sum_{j'=0}^{j-1}(\Delta n_{j+1} - \Delta n_j)(\Delta n_{j'+1} - \Delta n_{j'}) \\ & \cdot e^{-(\alpha+2ik)(j-j')a}(1 + m\cos\omega_m t) \\ & \cdot \left\{1 + m\cos\omega_m\left[t - \frac{2(j-j')a}{V}\right]\right\} \\ & \cdot e^{i\left\{\phi\left[t - \frac{2(j-j')a}{V}\right] - \phi(t)\right\}} \\ & \cdot e^{i\delta\left\{\cos\Omega_m\left[t - \frac{2(j-j')a}{V}\right] - \cos\Omega_m t\right\}} \end{aligned} \tag{8}$$

where, s_d is the responsivity of the detector (photocurrent $= s_d \times$ power), I_0 is the incident power. We have neglected the terms due to the beating of the scattered light with itself which are fourth order in $\Delta n(z)$.

We use (7) and (8) to obtain the current autocorrelation

$$C(\tau) = \langle i_d(t)i_d(t+\tau)\rangle. \tag{9}$$

By neglecting the small correction terms (second and fourth order in $\Delta n(z)$) to the dc, signal at ω_m, and signal at $2\omega_m$, we have

$$\begin{aligned} C(\tau) = {} & s_d^2 I_0^2 e^{-2\alpha L}\left\{\left(1 + \frac{m^2}{2}\right)^2 + 2m^2\cos\omega_m\tau\right. \\ & \left. + \frac{m^4}{8}\cos 2\omega_m\tau + 2\mathrm{Re}\langle g(t)g^*(t+\tau)\rangle\right\}. \end{aligned} \tag{10}$$

In evaluating $\langle g(t)g^*(t+\tau)\rangle$, one encounters: 1) fourth-order correlation functions which are evaluated using Wick's theorem

$$\begin{aligned} \langle\Delta n_1\Delta n_2\Delta n_3\Delta n_4\rangle = {} & \langle\Delta n_1\Delta n_2\rangle\langle\Delta n_3\Delta n_4\rangle \\ & + \langle\Delta n_1\Delta n_3\rangle\langle\Delta n_2\Delta n_4\rangle \\ & + \langle\Delta n_1\Delta n_4\rangle\langle\Delta n_2\Delta n_3\rangle \end{aligned} \tag{11}$$

and

$$\begin{aligned} & \left\langle e^{i\left\{\phi\left[t - \frac{2(j-j')a}{v}\right] - \phi(t) - \phi\left[t+\tau - \frac{2(l-l')a}{v}\right] + \phi(t+\tau)\right\}}\right\rangle \\ & = \exp\left\{-\frac{2}{\tau_c}\left[\frac{a}{c}(j - j' + l - l')\right.\right. \\ & - \min\left(\frac{2(l-l')a}{v}, \tau + \frac{2(j-j')a}{v}\right) \\ & \left.\left. + \min\left(\tau, \frac{2(l-l')a}{v}\right)\right]\right\} \end{aligned} \tag{12}$$

2) the factors due to AM and phase modulations

$$\begin{aligned} & \left\langle(1 + m\cos\omega_m t)[1 + m\cos\omega_m(t+\tau)]\right. \\ & \cdot\left[1 + m\cos\omega_m\left(t - \frac{2x}{v}\right)\right]\left[1 + m\cos\omega_m\left(t + \tau - \frac{2x}{v}\right)\right] \\ & \cdot\exp\left\{i\delta\left[\cos\Omega_m\left(t - \frac{2x}{v}\right) - \cos\Omega_m t - \cos\Omega_m\right.\right. \\ & \left.\left.\left.\cdot\left(t + \tau - \frac{2x}{v}\right) + \cos\Omega_m(t+\tau)\right]\right\}\right\rangle \\ & = \left[1 + \frac{m^4}{8} + m^2\cos\omega_m\tau + \frac{m^4}{8}\cos 2\omega_m\tau\right] \\ & \cdot J_0\left[4\delta\sin\left(\frac{\Omega_m\tau}{2}\right)\sin(k_m x)\right] \end{aligned} \tag{13}$$

where, $k_m = \Omega_m/v$, J_0 is the zeroth order Bessel function; 3) the quadruple integrations over the fiber length that is approximated using the inequalities $v\tau_c >> 1/k >> z_R$ ($v\tau_c \sim 1$ m, $1/k \sim 2000$ Å, $z_R \sim 100$ Å in practical systems).

Lengthy but straightforward algebra leads to

$$\begin{aligned} C(\tau) = {} & s_d^2 I_0^2 e^{-2\alpha L}\left\{\left(1 + \frac{m^2}{2}\right)^2 + 2m^2\cos\omega_m\tau\right. \\ & + \frac{m^4}{8}\cos 2\omega_m\tau + 4S^2(\alpha L_{\mathrm{eff}}). \end{aligned}$$

$$e^{-2\tau/\tau_c}\left[1+\frac{m^4}{8}+m^2\cos\omega_m\tau+\frac{m^4}{8}\cos 2\omega_m\tau\right] \cdot [f(\delta)+\cdots]\Bigg\} \tag{14}$$

where

$$S=\frac{\beta^2 k^2 z_R}{n_0^2\alpha} \tag{15}$$

$$L_{\text{eff}}=L-\frac{1}{2\alpha}\left(1-e^{-2\alpha L}\right) \tag{16}$$

$$f(\delta)=\sum_{k=0}^{\infty}(-1)^k\left[\frac{(2k)!}{2^k(k!)^3}\right]^2\delta^{2k} \tag{17}$$

and the omitted terms are higher Fourier components of $\Omega_m\tau$.

The spectral density function $W(\Omega)$ of the detected photocurrent at frequency Ω is obtained using the Wiener-Khintchine theorem

$$W(\Omega)=\frac{2}{\pi}\int_0^{\infty}C(\tau)\cos\Omega\tau\,d\tau \tag{18}$$

and is evaluated using (14)

$$\begin{aligned} W(\Omega)= s_d^2 I_0^2 e^{-2\alpha L}\Bigg\{&\left(1+\frac{m^2}{2}\right)^2\delta(\Omega)+2m^2\delta(\Omega-\omega_m)\\ &+\frac{m^4}{8}\delta(\Omega-2\omega_m)\\ &+4S^2(\alpha L_{\text{eff}})f(\delta)\Bigg[\left(1+\frac{m^4}{8}\right)\frac{\tau_c/\pi}{1+\frac{1}{4}\Omega^2\tau_c^2}\\ &+m^2\frac{\tau_c/2\pi}{1+\frac{1}{4}(\Omega-\omega_m)^2\tau_c^2}+\frac{m^4}{8}\frac{\tau_c/2\pi}{1+\frac{1}{4}(\Omega-2\omega_m)^2\tau_c^2}+\cdots\Bigg\} \end{aligned} \tag{19}$$

where: 1) the "delta" functions $\delta(\Omega)$, $\delta(\Omega-\omega_m)$, and $\delta(\Omega-2\omega_m)$ terms represent the dc, signal at ω_m, and signal at $2\omega_m$, respectively; 2) the $\tau_c/[1+1/4(\Omega-\omega_m)^2\tau_c^2]$ term represents the Lorentzian noise spectrum straddling the signal at ω_m etc.; 3) the omitted terms are the noise spectra around the frequencies $n\Omega_m+j\omega_m$, $n=1, 2, 3, \cdots$, and $j=\pm1, \pm2$. From (19), the noise to signal (at $\Omega=\omega_m$) power ratio is given by

$$\frac{\text{Noise}}{\text{Signal}}=2S^2(\alpha L_{\text{eff}})f(\delta)\int_{\omega_m-\frac{\Delta\Omega}{2}}^{\omega_m+\frac{\Delta\Omega}{2}}U(\Omega)\,d\Omega \tag{20}$$

where, $\Delta\Omega$ is the bandwidth of the detection circuit and $U(\Omega)$ is the (unity) normalized Lorentzian

$$U(\Omega)=\frac{\tau_c/2\pi}{1+\frac{1}{4}(\Omega-\omega_m)^2\tau_c^2}=\frac{\frac{2}{\pi\Delta\Omega_{1/2}}}{1+\frac{4(\Omega-\omega_m)^2}{(\Delta\Omega_{1/2})^2}} \tag{21}$$

and $\Delta\Omega_{1/2}=4/\tau_c$ is twice the FWHM optical linewidth of the laser output.

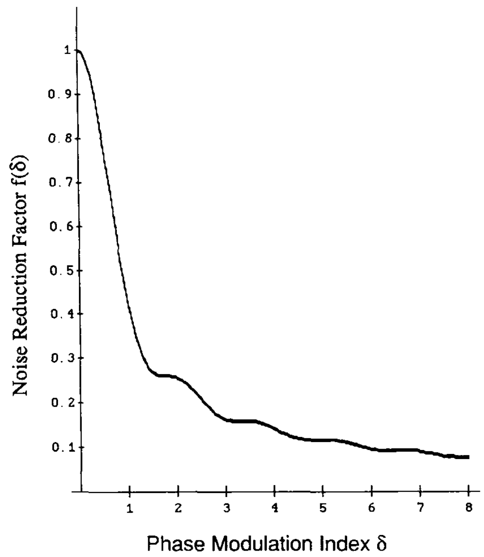

Fig. 3. A plot of $f(\delta)$ versus δ (using Mathematica). The first 50 terms of (17) are included. $f(\delta)\approx 0.1$ at $\delta=6$.

It is interesting to obtain an estimate of the noise to signal ratio in a practical system (e.g., fiber-optic cable TV system) without the phase modulation at Ω_m, i.e., $f(\delta)=1$. Using experimental data from reference [2], $\alpha=0.4$ dB/km, $2\pi\tau_c=10^{-7}$ s, and $S=6\times10^{-4}$, we calculate using (20) at $\Omega=\omega_m$ a noise to signal ratio of -72.4 dB for $L=20$ km and $\Delta\Omega=6$ MHz (per channel). This phase-to-intensity conversion noise, the largest noise source in the systems under study, limits the ultimate noise-to-signal performance.

With phase modulation at Ω_m, the noise level is reduced by the factor $f(\delta)$ given by (17). A plot of $f(\delta)$ vs δ is shown in Fig. 3, where one can see that the noise level can be reduced by as much as 10 dB at $\delta\approx6$. Physically, it means that an external phase modulation at high frequency generates sidebands and effectively spreads the intensity noise to all sidebands. Consequently, the noise level around the signal (ω_m) is reduced.

Although our analysis is specialized to the case of a fiber it is clear that any optical system in which interference of a laser output converts phase noise to amplitude noise will benefit from a pre-phase modulation of the optical carrier. In this paper, the analysis has been conducted under the condition that $\Omega_m >> \omega_m$, but we would like to point out that mathematically the same reduction in noise can still be achieved under a less restrictive condition that

$$|n\Omega_m+j\omega_m| >> \Omega_{1/2}=4/\tau_c,\quad j=\pm1,\pm2,\pm3;\quad \text{and}\quad n=\pm1,\pm2,\pm3,\cdots. \tag{22}$$

For a real system that transmits many AM signals at different frequencies (e.g., a typical fiber-optic cable TV system carrying many channels, bandwidth 100–550 MHz, 6 MHz per channel), the only way for (22) to be satisfied for all ω_m's is when $\Omega_m >> \omega_m$ (practically $\Omega_m > 3\omega_m$ is fine). It might be instructive to compare our phase modulation scheme to those employing "low-coherence lasers." The latter usually achieve their state of low coherence by random phase modulation. If the spectrum of this modulation is comparable or smaller than the information bandwidth there is no improvement in the detected intensity noise. To get a reduction of the noise it is necessary that the coherence time be much shorter than the inverse of the information bandwidth. But this is essentially what is achieved by our scheme where $\Omega_m >> \omega_m$. The effect of fiber chromatic dispersion on the heavily phase-chirped signal can be avoided by choosing to work at the laser wavelength where the fiber has zero dispersion.

In conclusion, the intensity fluctuation spectral density due to Rayleigh scattering in long haul fiber-optic communication systems can be reduced substantially by external phase modulation at very high frequencies.

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